

# Weighted ROC analysis

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## 1 Introduction

In binary classification, we are given  $n$  observations. For each observation  $i \in \{1, \dots, n\}$  we have an input/feature  $x_i \in \mathcal{X}$  and output/label  $y_i \in \{-1, 1\}$ . For example, say that  $\mathcal{X}$  is the space of all photographs, and we want to find a binary classifier that predicts whether a particular photograph  $x_i$  contains a car ( $y_i = 1$ ) or does not contain a car ( $y_i = -1$ ).

In weighted binary classification we also have observation-specific weights  $w_i \in \mathbb{R}_+$  which are the cost of making an error in predicting that observation. Thus the goal is to find a classifier  $c : \mathcal{X} \rightarrow \{-1, 1\}$  that minimizes the weighted zero-one loss on a set of test data

$$\underset{c}{\text{minimize}} \sum_{i \in \text{test}} I[c(x_i) \neq y_i] w_i, \quad (1)$$

where  $I$  is the indicator function that is 0 for a correct prediction, and 1 otherwise.

Instead of directly learning a classification function  $c$ , binary classifiers often instead learn a score function  $f : \mathcal{X} \rightarrow \mathbb{R}$ . Large values are more likely to be positive  $y_i = 1$  and small values are more likely to be negative. One way of evaluating such a model is by using the weighted Receiver Operating Characteristic (ROC) curve, as explained in the next section.

## 2 Weighted ROC curve

Let  $\hat{y}_i = f(x_i) \in \mathbb{R}$  be the predicted score for each observation  $i \in \{1, \dots, n\}$ , let  $\mathcal{I}_1 = \{i : y_i = 1\}$  be the set of positive examples and let  $\mathcal{I}_{-1} = \{i : y_i = -1\}$  be the set of negative examples. Then the total positive weight is  $W_1 = \sum_{i \in \mathcal{I}_1} w_i$  and the total negative weight is  $W_{-1} = \sum_{i \in \mathcal{I}_{-1}} w_i$ .

For any threshold  $\tau \in \mathbb{R}$ , define the thresholding function  $t_\tau : \mathbb{R} \rightarrow \{-1, 1\}$  as

$$t_\tau(\hat{y}) = \begin{cases} 1 & \text{if } \hat{y} \geq \tau \\ -1 & \text{if } \hat{y} < \tau. \end{cases} \quad (2)$$

We define the weighted false positive count as

$$\text{FP}(\tau) = \sum_{i \in \mathcal{I}_{-1}} I[t_\tau(\hat{y}_i) \neq -1] w_i \quad (3)$$

and the weighted false negative count as

$$\text{FN}(\tau) = \sum_{i \in \mathcal{I}_1} I[t_\tau(\hat{y}_i) \neq 1] w_i. \quad (4)$$

We define the weighted false positive rate as

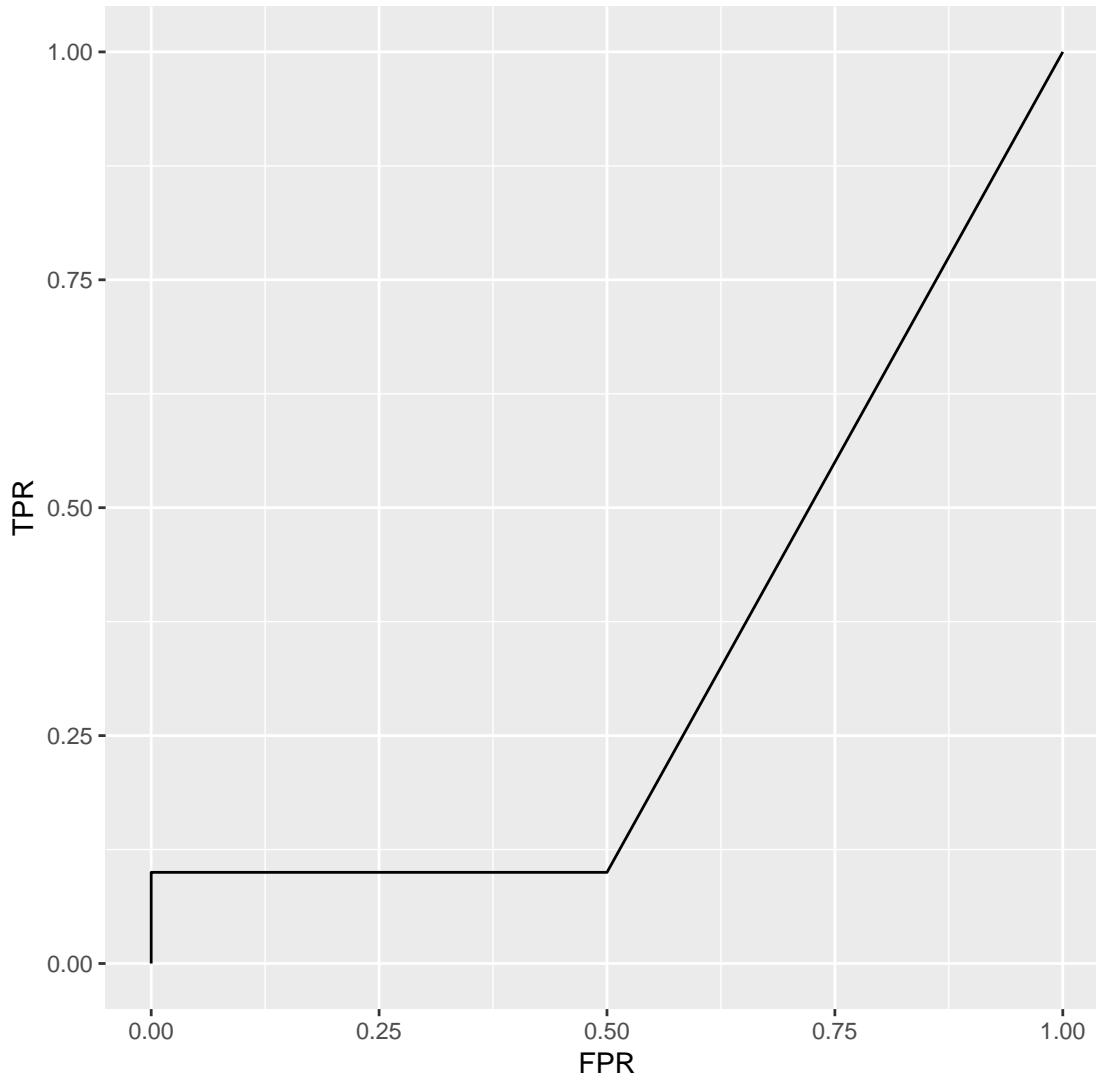
$$\text{FPR}(\tau) = \frac{1}{W_{-1}} \sum_{i \in \mathcal{I}_{-1}} I[t_\tau(\hat{y}_i) \neq -1] w_i \quad (5)$$

and the weighted true positive rate as

$$\text{TPR}(\tau) = \frac{1}{W_1} \sum_{i \in \mathcal{I}_1} I[t_\tau(\hat{y}_i) = 1] w_i. \quad (6)$$

A weighted ROC curve is drawn by plotting  $\text{FPR}(\tau)$  and  $\text{TPR}(\tau)$  for all thresholds  $\tau \in \mathbb{R}$ . It can be computed and plotted using the R code

```
> y <- c(-1, -1, 1, 1, 1)
> w <- c(1, 1, 1, 4, 5)
> y.hat <- c(1, 2, 3, 1, 1)
> library(WeightedROC)
> tp.fp <- WeightedROC(y.hat, y, w)
> library(ggplot2)
> ggplot()+
+   geom_path(aes(FPR, TPR), data=tp.fp)+
+   coord_equal()
```



### 3 Weighted AUC

The Area Under the Curve (AUC) may be computed using the R code

```
> WeightedAUC(tp.fp)
```

```
[1] 0.325
```