

Package ‘unifed’

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Title The Unifed Distribution

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Description Probability functions and family for `glm()` of the unifed distribution (Quijano Xacur, 2018; <arXiv:1812.00251>).

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R topics documented:

<code>dirwin.hall</code>	1
<code>dunifed</code>	2
<code>summary_unifed_glm</code>	4
<code>unifed</code>	4
<code>unifed.deviance</code>	5
<code>unifed.kappa</code>	6
<code>unifed.mle</code>	8
<code>unifed.stan</code>	9
<code>unifed.stan.path</code>	10
<code>unifed.varf</code>	10

 dirwin.hall

Irwin-Hall density

Description

Irwin-Hall density

Usage

```
dirwin.hall(x, n, log = FALSE)
```

Arguments

x	A number between 0 and n.
n	Number of uniform distributions in the unit interval to sum.
log	If it evaluates to TRUE it returns the log of the density instead of the density.

Details

Gives the density of the Irwin-Hall distribution. It is the density of the sum of n uniform distributions on the interval $(0,1)$.

$$h(y; n) = \frac{1}{(n-1)!} \sum_{k=0}^{\lfloor y \rfloor} (-1)^k \binom{n}{k} (y-k)^{n-1}$$

where $x \in [0, 1]$ and n is a positive integer.

This function is not numerically stable. The examples have some cases of this.

Examples

```
dirwin.hall(2, 5)

# Numerically unstable example
# Run the following one after the other
# See how it goes from positive to negative (which means overflowing )
dirwin.hall(35, 50)
dirwin.hall(36, 50)
dirwin.hall(37, 50)
dirwin.hall(38, 50)
```

`dunifed`*The unified distribution*

Description

Density, distribution function, quantile function and random generation for the unified distribution.

Usage`dunifed(x, theta)``unifed.lcdf(x, theta)``punifed(q, theta)``qunifed(p, theta)``runifed(n, theta)`**Arguments**

<code>x</code>	A vector of quantiles. They must be numbers between 0 and 1.
<code>theta</code>	The value of the canonical parameter. It must be of length one.
<code>q</code>	A vector of quantiles.
<code>p</code>	A vector of probabilities.
<code>n</code>	number of observations

Value

`dunifed` gives the density function.

`unifed.lcdf` returns the log of the cumulative distribution function of the unified.

`punifed` gives the distribution function.

`qunifed` gives the quantile function.

`runifed` generates random observations.

References

Quijano Xacur, Oscar Alberto (2018). The Unified Distribution. ArXiv. <http://arxiv.org/abs/1812.00251>.

Examples

```
dunifed( c(0.1,0.3,0.7), 10)
```

```
x <- c(0.3,0.6,0.9)
unifed.lcdf(x,5)
```

```
x <- c(0.1,0.4,0.7,1)
punifed(x,-5)
```

```
p <- 1:9/10
qunifed(p,5)
```

```
runifed(20,-3.3)
```

summary_unifed_glm *Summarizing Generalized Linear Model Fits*

Description

Wrapper function for summary.glm.

Usage

```
summary_unifed_glm(object, ...)
```

Arguments

object an object of class "glm".

... Other arguments for stats::summary.glm.

This wrapper function was created in order to automatically set to 1 the dispersion parameter of a fitted unifed GLM. When the package is loaded the summary method of the glm class is rewritten using this function.

unifed

*Family object for the unifed distribution***Description**

Family object for the unifed distribution

Usage

```
unifed(link = "logit", ...)
quasiunifed(link = "logit", ...)
unifed.canonical.link()
```

Arguments

`link` a specification for the model link function. This can be a name/expression, a literal character string, a length-one character vector or an object of class "link-glm" (such as generated by `make.link()`) provided it is not specified via one of the accepted names. The `unifed` family accepts the links (as names) 'canonical', 'logit', 'probit', 'cloglog' and 'cauchit'.

`...` Optional `tol` and `maxit` arguments for `unifed.unit.deviance`.

Details

The link 'canonical' is not part of the standard names accepted by `make.link()` from the `stats` package. It corresponds to the canonical link function for the unifed distribution, which is the inverse of the derivative of its cumulant generator. There is no explicit formula for it. The function `unifed.kappa.prime.inverse()` implements it using the Newton-Raphson method.

Value

`unifed` returns a family object for using the unifed distribution with the `glm` function.

The `quasiunifed` family differs from the `unifed` only in that the dispersion parameter is not fixed to one.

An object of class "link-glm".

References

- Jørgensen, Bent (1992). The Theory of Exponential Dispersion Models and Analysis of Deviance. Instituto de Matemática Pura e Aplicada, (IMPA), Brazil.
- Wedderburn, R. W. M. (1974). Quasi-likelihood functions, generalized linear models, and the Gauss—Newton method. *Biometrika*. **61** (3): 439–447.
- McCullagh, Peter; Nelder, John (1989). *Generalized Linear Models* (second ed.). London: Chapman and Hall.

See Also

Gamma `unifed.kappa.prime.inverse`
[make.link](#)

`unifed.deviance` *Deviance of the unifed distribution*

Description

Deviance of the unifed distribution

Usage

```
unifed.deviance(y.v, mu.v, wt = 1, ...)
unifed.unit.deviance(y, mu, tol = 1e-07, maxit = 50)
```

Arguments

<code>y.v</code>	A numeric vector with values between 0 and 1
<code>mu.v</code>	A numeric vector with values between 0 and 1
<code>wt</code>	(default value: 1) The weight vector. It contains the weight of each observation. It must contain positive integers only.
<code>...</code>	Additional parameters of <code>unifed.kappa.prime.inverse.one</code>
<code>y</code>	A vector with values between 0 and 1.
<code>mu</code>	A vector with values between 0 and 1.
<code>tol</code>	Tolerance level for the Newton-Raphson algorithm for computing the inverse of the derivative of the cumulant generator of the family.
<code>maxit</code>	Maximum number of iterations for the Newton-Raphson algorithm for computing the inverse of the derivative of the cumulant generator of the family.

Details

`unifed.unit.deviance` uses the following expression for the deviance of regular exponential dispersion families

$$d(y, \mu) = 2 [y\{\dot{\kappa}^{-1}(y) - \dot{\kappa}^{-1}(\mu)\} - \kappa(\dot{\kappa}^{-1}(y)) + \kappa(\dot{\kappa}^{-1}(\mu))]$$

$\dot{\kappa}^{-1}$ is computed with the function `unifed.kappa.prime.inverse` from this package.

Value

`unifed.deviance` returns the deviance of a GLM with a unified response distribution. This is

$$D(\mathbf{y}, \boldsymbol{\mu}) = \sum_{i=1}^m w_i d(y_i, \mu_i)$$

Where $d(y_i, \mu_i)$ is the unit deviance of the unified distribution between the i -th entry of \mathbf{y} and $\boldsymbol{\mu}$. w_i is the i -th entry of the weight vector. `unifed.unit.deviance` is used to get the value of d .

`unifed.unit.deviance`

<code>unifed.kappa</code>	<i>Cumulant generator of the unified distribution</i>
---------------------------	---

Description

Cumulant generator of the unified distribution

Usage

`unifed.kappa(theta)`

`unifed.kappa.prime(theta)`

`unifed.kappa.double.prime(theta)`

`unifed.kappa.prime.inverse(mu, ...)`

`unifed.kappa.prime.inverse.one(mu, tol = 1e-07, maxit = 1e+07)`

Arguments

<code>theta</code>	A numeric vector.
<code>mu</code>	A vector of numbers between 0 and 1
<code>...</code>	Other parameters of <code>unifed.kappa.prime.inverse.one</code>
<code>tol</code>	Tolerance level. The algorithm stops if the proportional difference between the new and old value of an iteration is less or equal than this number.
<code>maxit</code>	Maximum number of iterations of the algorithm to look for convergence.

Details

The cumulant generator of the unified distribution is defined as

$$\kappa(\theta) = \begin{cases} \log\left(\frac{e^\theta - 1}{\theta}\right) & \text{if } \theta \neq 0 \\ 0 & \text{if } \theta = 0 \end{cases} .$$

`unifed.kappa.prime.inverse.one` uses the Newton-Raphson method for finding the inverse of `unifed.kappa.prime` for a single value.

Value

`unifed.kappa` returns a vector that contains the cumulant generator of the unified distribution applied to each element of `theta`.

`unifed.kappa.prime` returns a vector that contains the derivative of the cumulant generator of the unified distribution for each element of `theta`.

`unifed.kappa.double.prime` returns a vector that contains the second derivative of the cumulant generator of the unified distribution for each element of `theta`.

`unifed.kappa.prime.inverse` returns a vector with `unifed.kappa.prime.inverse.one` evaluated at every entry of `mu`.

`unifed.kappa.prime.inverse.one` if the tolerance level is reached within `maxit` iterations, the function returns the value of the last iteration. Otherwise it returns `NA`.

References

Quijano Xacur, Oscar Alberto (2018). The Unified Distribution. ArXiv. <http://arxiv.org/abs/1812.00251>.

Jørgensen, Bent (1997). The Theory of Dispersion Models. Chapman & Hall, London.

Examples

```
unifed.kappa(1)
unifed.kappa(-5:5)
```

```
unifed.kappa.prime(4.5)
```

```
unifed.kappa.double.prime(0)
```

```
unifed.kappa.prime.inverse(0.5)
unifed.kappa.prime.inverse(c(0.1, 0.7, 0.9))
```

`unifed.mle`*Maximum Likelihood Estimate for the unified distribution*

Description

Maximum Likelihood Estimate for the unified distribution

Usage

```
unifed.mle(x)
```

Arguments

`x` A numeric vector with values in the interval $[0,1]$.
 Computes the maximum likelihood estimator of the canonical parameter of the unified distribution. It is assumed that the elements of `x` come from independent and identically distributed unified random variables.

Examples

```
a.unifed.sample <- runifed(1000,10)
theta.mle <- unifed.mle(a.unifed.sample)
```

unifed.stan

Stan functions for working with the unified distribution

Description

Stan functions for working with the unified distribution

Details

A script with stan functions of the unified is provided. The script can be included in stan code. The full path to the script can be obtained with the function `unifed.stan.path`. The following list are the names of functions that take one real value:

```
real unifed_kappa(real theta) Computes the cumulant generator of the unified distribution.
real unifed_kappa_prime(real theta) Computes the first derivative of the cumulant generator.
real unifed_kappa_double_prime(real theta) Computes the second derivative of the cumulant generator.
real unifed_lpdf(real x, real theta) Computes the logarithm of the probability density function of a unified distribution. theta is the value of the canonical parameter of the unified and x if the value where the density is evaluated.
real unifed_quantile(real p, real theta) Returns the p-th quantile of a unified distribution with canonical parameter theta.
real unifed_rng(real theta) Returns a simulated value of a unified distribution with canonical parameter theta.
real unifed_lcdf(real x, real theta) Computes the logarithm of the cumulative density function of a unified distribution. theta is the value of the canonical parameter of the unified and x if the value where the density is evaluated.
real unifed_kappa_prime_inverse(real mu) Returns the inverse of the derivative of the unified cumulant generator
real unifed_unit_deviance(real y, real mu) Unit deviance function of the unified.
```

The following functions take vectors as arguments

`vector unifed_kappa_v (vector theta)` Vectorized version of `unifed_kappa`.

`vector unifed_kappa_prime_inverse_v (vector mu)` Vectorized version of `unifed_kappa_prime_inverse`.

`void unifed_glm_lp (vector y, vector theta, vector weights)` Adds to the Log Probability Accumulator the logarithm of the likelihood function of a GLM with observed response `y`, estimated canonical parameter `theta` and weights `weights`.

`unifed.stan.path` *Unifed Stan function paths*

Description

The `unifed.stan` provided by the file contains functions for using the `unifed` distribution in `stan`. The file can be included (with `#include`) inside the functions block of a `stan` program or its contents can be copied and pasted.

Usage

`unifed.stan.path()`

`unifed.stan.folder()`

Value

The full path to the `unifed.stan` file provided by the package.

`unifed.stan.folder` returns a string containing the path to the folder containing the `unifed.stan` file. This can be used as the `isystem` parameter in `stan` functions.

`unifed.varf` *Variance function of the unifed distribution*

Description

Variance function of the `unifed` distribution

Usage

`unifed.varf (mu)`

Arguments

`mu` A vector with numbers between 0 and 1.

Value

It returns `unifed.kappa.double.prime (unifed.kappa.prime.inverse (mu))`.